

Nonperturbative corrections to $B \rightarrow X_s \ell^+ \ell^-$ with phase space restrictions

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Abstract

We study nonperturbative corrections up to $\mathcal{O}(1/m_b^3)$ in the inclusive rare B decay $B \rightarrow X_s \ell^+ \ell^-$ by performing an operator product expansion. The values of the matrix elements entering at this order are unknown and introduce uncertainties into physical quantities. Imposing a phase space cut to eliminate the $c\bar{c}$ resonances we find that the $\mathcal{O}(1/m_b^3)$ corrections introduce an $\mathcal{O}(10\%)$ uncertainty in the measured rate. We also find that the contributions arising at $\mathcal{O}(1/m_b^3)$ are comparable to the ones arising at $\mathcal{O}(1/m_b^2)$ over the entire region of phase space.

By 1995 CLEO had measured the rates for both the exclusive decay $B \rightarrow K^* \gamma$ [1] and for the inclusive process $B \rightarrow X_s \gamma$ [2], marking the advent of experimental studies of penguin-mediated B decays. Such processes arise in the standard model at the one loop level. Physics from beyond the standard model may appear in the loop with an amplitude comparable to the standard model amplitude, thereby making such rare decays an excellent testing ground for standard model extensions. Of course, efforts to detect deviations from the standard model are frustrated by uncertainties in standard model predictions.

The decay $B \rightarrow X_s \ell^+ \ell^-$, though it has not yet been observed [3], has garnered recent interest because of its sensitivity to new physics not contributing to the decay $B \rightarrow X_s \gamma$. The $\mathcal{O}(1/m_b^2)$ nonperturbative corrections to $\Gamma(B \rightarrow X_s \ell^+ \ell^-)$ have been previously calculated [4,5]. We extend that study to calculate the $\mathcal{O}(1/m_b^3)$ corrections for massless leptons in the final state ($\ell = e, \mu$) following the similar calculations for semileptonic B decays [6] and the decay $B \rightarrow X_s \gamma$ [7]. We use the standard effective Hamiltonian mediating the $b(p_b) \rightarrow s(p_s) + \ell^+(p_+) + \ell^-(p_-)$ transition obtained from integrating out the top quark and the weak bosons. It is given by

$$\mathcal{H}_{eff}(b \rightarrow s \ell^+ \ell^-) = -4 \frac{G_F}{\sqrt{2}} |V_{ts}^* V_{tb}| \sum_{i=1}^{10} C_i(\mu) O_i(\mu). \quad (1)$$

The operator basis $\{O_i\}$ can be found in the literature [8].

The Wilson coefficients $\{C_i\}$ at the scale $\mu \sim m_b$ are known in the next to leading log approximation [9,10]. For consistency with the literature we have defined two effective Wilson coefficients: $C_7^{eff} \equiv C_7 - C_5/3 - C_6$ and C_9^{eff} . The latter contains the operator mixing of O_{1-6} into O_9 as well as the one loop matrix elements of $O_{1-6,9}$ [9,10]. The full analytic expression for C_9^{eff} is quite lengthy and may be found in [10].

For the branching ratio at the parton level we find, in agreement with previous calculations [5,8],

$$\begin{aligned} \frac{\mathcal{B}_{\text{parton}}}{\mathcal{B}_0} = & -\frac{32}{9} \left(4 + 3 \log \left(\frac{4m_l^2}{M_B^2} \right) \right) C_7^{eff^2} + \frac{2}{3} C_{10}^2 + 128 C_7^{eff} \int_0^{\frac{1}{2}} dx_0 x_0^2 C_9^{eff}(x_0) \\ & + \frac{32}{3} \int_0^{\frac{1}{2}} dx_0 (3x_0^2 - 4x_0^3) |C_9^{eff}(x_0)|^2 \end{aligned} \quad (2)$$

where $x_0 \equiv E_0/m_b$ is the rescaled final state parton energy, and \mathcal{B}_0 is the normalization factor

$$\mathcal{B}_0 = \mathcal{B}_{sl} \frac{3\alpha^2}{16\pi^2} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{1}{f(\hat{m}_c) \kappa(\hat{m}_c)}. \quad (3)$$

Here \mathcal{B}_{sl} is the measured semileptonic branching ratio, $f(\hat{m}_c)$ is the phase space factor for $\Gamma(B \rightarrow X_c \ell \bar{\nu})$

$$f(\hat{m}_c) = 1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - \hat{m}_c^8 - 24\hat{m}_c^4 \log(\hat{m}_c), \quad (4)$$

and $\kappa(\hat{m}_c)$ accounts for the $\mathcal{O}(\alpha_s)$ QCD correction and the leading power corrections. The complete expression for $\kappa(\hat{m}_c)$ may be found in [4]. As alluded to above, the analytic form of C_9^{eff} is sufficiently complicated that we must resort to numerical integrations.

The procedure for calculating nonperturbative contributions to heavy hadron decays has been discussed in great detail in the literature [11,12], and we give only a short review here. The differential rate is proportional to the product of the lepton tensor $L_{\mu\nu}$ and the hadron tensor $W^{\mu\nu}$, which for the process in question may be written as

$$d\Gamma = \frac{1}{2M_B} \frac{G_F^2 \alpha^2}{2\pi^2} |V_{ts}^* V_{tb}|^2 d\Pi \left(L_{\mu\nu}^L W^{L\mu\nu} + L_{\mu\nu}^R W^{R\mu\nu} \right) \quad (5)$$

where Π denotes the three body phase space. The hadron tensor $W^{\mu\nu}$ is related via the optical theorem to the imaginary part of the forward scattering matrix element $W^{\mu\nu} = 2\text{Im}T^{\mu\nu}$ where

$$T_{\mu\nu}^{L(R)} = -i \int d^4x e^{-iq \cdot x} \langle B | T \{ J_\mu^{L(R)\dagger}(x), J_\nu^{L(R)}(0) \} | B \rangle \quad (6)$$

and the spin-summed tensor $L_{\mu\nu}$ for massless leptons is

$$L_{\mu\nu}^{L(R)} = 2 \left[p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - g^{\mu\nu} p_+ \cdot p_- \mp i\epsilon^{\mu\nu\alpha\beta} p_{+\alpha} p_{-\beta} \right]. \quad (7)$$

In (6) J^μ denotes the current mediating this transition, and is given by

$$J_{L(R)}^\mu = \bar{s} \left[R\gamma^\mu \left(C_9^{eff} \mp C_{10} + 2C_7^{eff} \frac{\hat{q}}{\hat{q}^2} \right) + 2\hat{m}_s C_7^{eff} \gamma^\mu \frac{\hat{q}}{\hat{q}^2} L \right] b \quad (8)$$

where $L(R) = \frac{1}{2}(1 \mp \gamma_5)$ are the usual left and right handed chiral projection operators, and $q \equiv (p_+ + p_-)$ is the dilepton momentum.

It has been shown in [11,12] that the time-ordered product in (6) can be expanded as an operator product expansion (OPE), given schematically by

$$-i \int d^4x e^{-iq \cdot x} T\{J^\dagger(x), J(0)\} \sim \frac{1}{m_b} \left[O_0 + \frac{1}{2m_b} O_1 + \frac{1}{4m_b^2} O_2 + \frac{1}{8m_b^3} O_3 + \dots \right], \quad (9)$$

where O_n represents a set of local operators of dimension $(3+n)$. In this study we include operators up to dimension six [7].

Matrix elements of dimension four operators vanish [11] at leading order in the $1/m_b$ expansion and matrix elements of dimension five operators may be parameterized by λ_1 and λ_2 [13]

$$\langle B(v) | \bar{h}_v \Gamma i D_\mu i D_\nu h_v | B(v) \rangle = M_B \text{Tr} \left\{ \Gamma P_+ \left(\frac{1}{3} \lambda_1 (g_{\mu\nu} - v_\mu v_\nu) + \frac{1}{2} \lambda_2 i \sigma_{\mu\nu} \right) P_+ \right\}, \quad (10)$$

where $P_+ = \frac{1}{2}(1 + \not{v})$ projects onto the effective spinor h_v , and Γ is an arbitrary Dirac structure.

Finally, the dimension six operators may be parameterized by the matrix elements of two local operators [6,14]

$$\begin{aligned} \frac{1}{2M_B} \langle B(v) | \bar{h}_v i D_\alpha i D_\mu i D_\beta h_v | B(v) \rangle &= \frac{1}{3} \rho_1 (g_{\alpha\beta} - v_\alpha v_\beta) v_\mu, \\ \frac{1}{2M_B} \langle B(v) | \bar{h}_v i D_\alpha i D_\mu i D_\beta \gamma_\delta \gamma_5 h_v | B(v) \rangle &= \frac{1}{2} \rho_2 i \epsilon_{\nu\alpha\beta\delta} v^\nu v_\mu \end{aligned} \quad (11)$$

and by matrix elements of two time-ordered products

$$\begin{aligned} \frac{1}{2M_B} \langle B(v) | \bar{h}_v (iD)^2 h_v i \int d^3x \int_{-\infty}^0 dt \mathcal{L}_I(x) | B(v) \rangle + h.c. &= \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b}, \\ \frac{1}{2M_B} \langle B(v) | \bar{h}_v \frac{1}{2} (-i\sigma_{\mu\nu}) G^{\mu\nu} h_v i \int d^3x \int_{-\infty}^0 dt \mathcal{L}_I(x) | B(v) \rangle + h.c. &= \frac{\mathcal{T}_3 + 3\mathcal{T}_4}{m_b}. \end{aligned} \quad (12)$$

The contributions from \mathcal{T}_{1-4} can most easily be incorporated by making the replacements [6]

$$\begin{aligned} \lambda_1 &\rightarrow \lambda_1 + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b} \\ \lambda_2 &\rightarrow \lambda_2 + \frac{\mathcal{T}_3 + 3\mathcal{T}_4}{3m_b} \end{aligned} \quad (13)$$

in the final analytic results. In addition, there is a contribution to the total rate from the four-fermion operator

$$O_{(V-A)}^{bs} = 16\pi^2 \left[\bar{b} \gamma^\mu L s \bar{s} \gamma^\nu L b (g_{\mu\nu} - v_\mu v_\nu) \right], \quad (14)$$

the matrix element of which we define as

$$\frac{1}{2M_B} \langle B | O_{(V-A)}^{bs} | B \rangle \equiv f_1. \quad (15)$$

Our analytic expression for the differential branching ratio agrees with the results presented in [4] up to $\mathcal{O}(1/m_b^2)$, and will be presented in detail elsewhere [16]. Here we restrict ourselves to numerical results. In Figure 1 we plot the differential branching ratio $\frac{1}{\mathcal{B}_0} \frac{d\mathcal{B}}{d\hat{q}^2}$, where $\hat{q} = q/m_b$. The solid line is the parton model result, the long-dashed line includes corrections up to $\mathcal{O}(1/m_b^2)$ and the short-dashed line incorporates typical $\mathcal{O}(1/m_b^3)$ corrections as well. In these plots we have used $\lambda_2 = 0.12 \text{ GeV}^2$ as indicated by the $B^* - B$ mass splitting, and $\lambda_1 = -0.19 \text{ GeV}^2$ [17]. For the $\mathcal{O}(1/m_b^3)$ matrix elements, whose values are unknown, we have chosen a generic size $|\rho_i|, |\mathcal{T}_i| \sim \Lambda_{\text{QCD}}^3 \sim (0.5 \text{ GeV})^3$ as suggested by dimensional analysis.

Compared to the parton model prediction the nonperturbative corrections are small over almost the entire range of \hat{q}^2 , and become large only near the $\hat{q}^2 \rightarrow 1$ endpoint. It is a well known feature that close to this endpoint the OPE (9) breaks down and the differential spectrum is determined by the shape function [18]. Once this spectrum is smeared with a weight function that varies slowly in the endpoint region the OPE should be convergent. However, as can be seen from Figure 1 the differential branching ratio diverges in the $\hat{q}^2 \rightarrow 1$ endpoint as

$$\left. \frac{1}{\mathcal{B}_0} \frac{d\mathcal{B}}{d\hat{q}^2} \right|_{\hat{q}^2 \rightarrow 1} \sim \frac{\rho_1}{1 - \hat{q}^2}, \quad (16)$$

yielding an unphysical logarithmic divergence in the integrated spectrum that is regulated by the s quark mass. This apparent problem is solved by an additional term that contributes only at the endpoint

$$\frac{d\Gamma}{d\hat{q}^2} = \frac{d\Gamma}{d\hat{q}^2}|_{\text{reg}} - 16 (C_{10}^2 + (2C_7^{\text{eff}} + C_9^{\text{eff}})^2) \delta(q^2 - 1) (\rho_1 \log(\hat{m}_s) - f_1), \quad (17)$$

where $\frac{d\Gamma}{d\hat{q}^2}|_{\text{reg}}$ is the function plotted in Figure 1. The $\log(\hat{m}_s)$ term multiplying the delta function removes the divergence mentioned above and the appearance of the four fermion operator has been discussed in the context of semileptonic B decays [19,20]. A more detailed discussion of these issues will be given elsewhere [16].

The long distance $c\bar{c}$ resonances in the $d\mathcal{B}/d\hat{q}^2$ spectrum must be cut out before the theory can be compared to measurements. Thus we investigate the importance of the non-perturbative corrections when we integrate only a fraction of phase space $\hat{q}^2 > \chi$. We define a partially integrated branching ratio

$$\mathcal{B}_\chi = \frac{1}{\mathcal{B}_0} \int_\chi^1 d\hat{q}^2 \frac{d\mathcal{B}}{d\hat{q}^2} \quad (18)$$

that depends on the size of the accessible phase space. Figure 2 shows the fractional correction to the integrated parton level rate from each of the nonperturbative parameters λ_i, ρ_i as a function of the minimum accessible dilepton invariant mass χ . In this plot we have chosen the same values for the nonperturbative parameters as in Figure 1. Over the

entire range of \hat{q}^2 the contributions from $\mathcal{O}(1/m_b^3)$ operators are comparable to the ones from $\mathcal{O}(1/m_b^2)$ operators. This indicates that this decay is unsuitable for extracting the matrix element λ_1 as has been suggested in [4]. This issue will be investigated more in [16]. Of course, the sizes of the ρ_i contributions shown here should not be taken as accurate indications of the actual size of the corrections, but rather as estimates of the uncertainty in the prediction. We see that for $\chi \sim 0.75$ the contribution from the ρ_1 matrix element is potentially of the same size as the parton model prediction. This is a clear signal that the OPE is no longer valid. Even at $\chi \sim 0.5$ the contribution is about 10%, which is very interesting in light of the fact that the CLEO search strategy for this decay imposes the cut [3] $\hat{q}^2 \geq \chi = (m_{\psi'} + 0.1 \text{ GeV})^2/m_b^2 = 14.33 \text{ GeV}^2/m_b^2 = 0.59$, where we have used $m_b = 4.9 \text{ GeV}$. Investigating this particular value of the cut in more detail we find the individual contributions to the integrated spectrum to be

$$\begin{aligned} \mathcal{B}_{0.59} = & 3.8 + 1.9 \left(\frac{\lambda_1}{m_b^2} + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b^3} \right) - 134.7 \left(\frac{\lambda_2}{m_b^2} + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{3m_b^3} \right) \\ & + 614.9 \frac{\rho_1}{m_b^3} + 134.7 \frac{\rho_2}{m_b^3} + 560.8 \frac{f_1}{m_b^3}. \end{aligned} \quad (19)$$

To estimate the uncertainty induced by the $\mathcal{O}(1/m_b^3)$ parameters we fix λ_i to the values given above, then randomly vary the magnitudes of the parameters ρ_i, \mathcal{T}_i and f_1 between $-(0.5 \text{ GeV})^3$ and $(0.5 \text{ GeV})^3$ as suggested by dimensional analysis. We also impose positivity of ρ_1 as indicated by the vacuum saturation approximation [21], and the constraint [6]

$$\rho_2 - \mathcal{T}_2 - \mathcal{T}_4 = \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{3/\beta_0} \frac{M_B^2 \Delta M_B (M_D + \bar{\Lambda}) - M_D^2 \Delta M_D (M_B + \bar{\Lambda})}{M_B + \bar{\Lambda} - \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{3/\beta_0} (M_D + \bar{\Lambda})} \quad (20)$$

derived from the ground state meson mass splittings $\Delta M_H = M_{H^*} - M_H$ ($H = B, D$). Here β_0 is the well known coefficient of the beta function $\beta_0 = 11 - \frac{2}{3}n_f$. Taking the 1σ deviation as a reasonable estimate of the uncertainties from $\mathcal{O}(1/m_b^3)$ contributions to the total rate at this cut, we again find the uncertainty to be at the 10% level. Relaxing the positivity constraint on ρ_1 enlarges the uncertainty to about 20%.

We have calculated the $\mathcal{O}(1/m_b^3)$ contributions to the differential spectrum $d\mathcal{B}/d\hat{q}^2$. We have found that the contributions of the new operators are small compared to the parton level spectrum except close to the endpoint $\hat{q}^2 \rightarrow 1$, where it is well known that the convergence of the OPE breaks down. Due to large numerical coefficients, however, the contributions from dimension six operators are comparable to the contributions from the dimension five operators. We have also investigated the uncertainties from the new nonperturbative operators on the total decay rate evaluated with a lower cut χ on the dilepton invariant mass. For $\chi \sim 0.59$, as proposed by CLEO, the uncertainties are around 10%. Increasing the value of χ rapidly increases the uncertainties on the partially integrated rate. At $\chi \sim 0.75$ this uncertainty is 100%, signalling that the convergence of the OPE has broken down.

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FIGURES

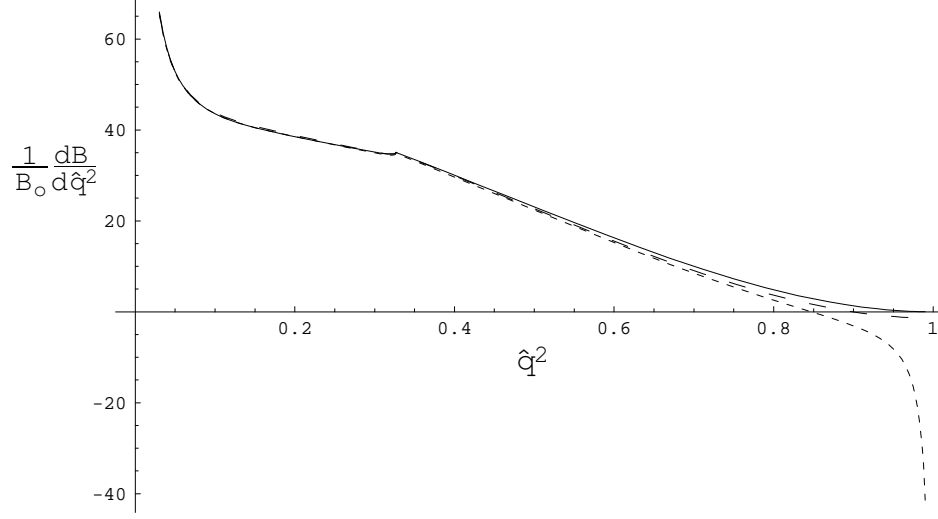


FIG. 1. The differential decay spectrum. The solid line shows the parton model prediction, the dashed line includes the $\mathcal{O}(1/m_b^2)$ corrections and the dotted line contains all corrections up to $\mathcal{O}(1/m_b^3)$.

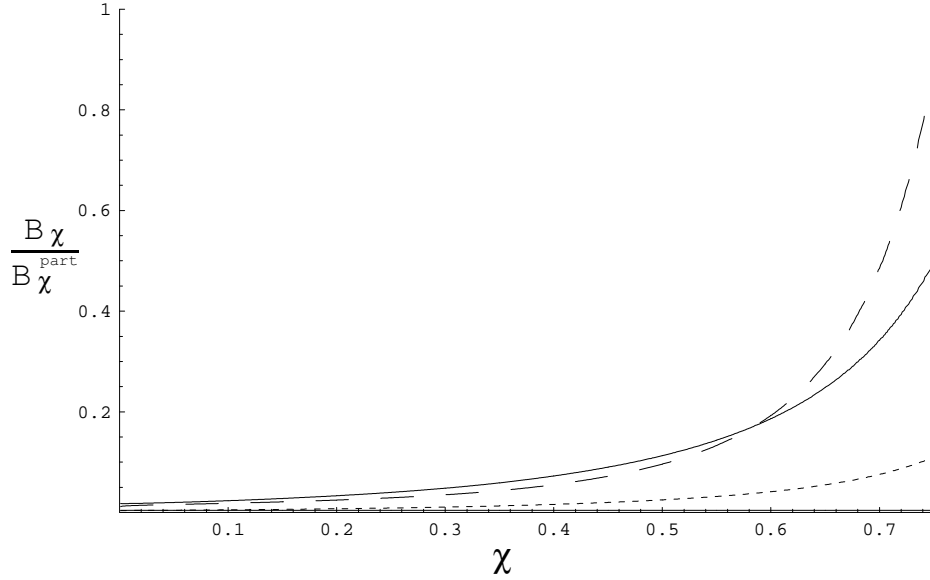


FIG. 2. The fractional contributions with respect to the parton model result from the higher dimensional operators. The solid, dashed and dotted lines correspond to the contributions from λ_2 , ρ_1 and ρ_2 , respectively. The contribution from λ_1 is too small to be seen.